A Heuristic Approach for Solving the Fixed Charge Transportation Problems

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Abstract

Most of researchers use the relaxed transportation problem proposed by (Balinski, 1961) to find approximate solution for the fixed charge transportation problem (FCTP). This approximated solution is considered as a lower limit for the optimal solution of FCTP. In this paper a heuristic approach has been developed to find an approximate solution used as a lower limit for the FCTP which is better than that is found by (Balinski, 1961). The same has been validated by applying the algorithm on 37 examples and testing for the significance of results. The algorithm is based on applying the Vogel approximation method on the relaxed transportation problem. In addition, an illustrative numerical example is given to show the simplicity of applying the proposed approach.

Keywords: *Transportation Problem, Fixed Charge, Heuristic Methods.*

Introduction

The fixed-charge problem deals with situations in which the activity incurs two types of costs: a fixed cost that must be incurred to start the activity and a variable cost that is directly proportional to the level of the activity. The fixed-charge transportation problem (FCTP) is a special case of the transportation problem where a fixed charge is associated with each route that can be opened, in addition to the variable transportation cost proportional to the amount of goods shipped. The objective is to select a distribution scheme with the minimum total cost. The FCTP is considered to be an NP-hard problem (Altassan, El-Sherbiny, & Sasidhar, 2013) and was first formulated in (Hirsch & Dantzig, 1954). In the literature, several algorithms have been proposed for solving this problem but the known exact ones are generally not very useful when a problem reaches a certain dimension. Therefore, some heuristic methods have been developed. Some of them are in (Adlakha & Kowalski, 2003; Adlakha, Kowalski, & Vemuganti, 2006; Cooper, 1975; Diaby, 1991; Gottlieb & Paulmann, 1998; Palekar, Karwan, & Zionts, 1990; Sun, Aronson, McKeown, & Drinka, 1998) .

An analytical branching method to solve the FCTP starting with a linear formulation of the problem that converges to an optimal solution by sequentially separating the fixed costs and finding a direction to improve the value of the objective function of the linear formulation has been proposed in (Adlakha, Kowalski, & Lev, 2010). An approximation for the lower bound of the FCTP is presented in (Adlakha, Kowalski, Wang, Lev, & Shen, 2014) and claimed that it is much superior to the lower bound developed in (Balinski, 1961). However, in the process, it was transformed to an NLP problem, which is computationally not simple.

The Fixed-charge Transportation Problem

The transportation model is a special case of the linear programming problem. It deals with transporting certain product from m sources to n destinations. The sources are production facilities with respictive capacities $a_1, a_2, \ldots, a_m$ and the destinations are warehouses with required levels of demands $b_1, b_2, \ldots, b_n$. The penality for transporting one unit of the given product from the source i to the destination j is c_{ij} . In the (FCTP) an additional fixed cost f_{ij} is assumed for opening the route (i, j) and the problem is to determine the amounts to be transported from all sources to all destinations such that the total transportation cost is minimized while satisfying both the supply limits and the demand requirements.

The mathematical model of the FCTP can be represented as follows:

$$Min \quad z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + f_{ij}y_{ij}) \qquad (1)$$

$$\text{s.t.} \quad \sum_{i=1}^{m} x_{ij} \geq b_j \quad \text{for } j = 1,\ldots,n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad \text{for } i = 1,\ldots,m \qquad (3)$$

$$\forall i, j \quad x_{ij} \geq 0$$

$$y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} \leq 0 \end{cases}$$

Where x_{ij} is the unknown quantity to be transported through the route (i, j).

The Proposed Algorithm

This algorithm gives a better approximate solution for the FCTP that given in (Balinski, 1961). The proposed algorithm is an improved version of that Vogel Approximation Method (VAM) that generally produces a approximate solution for the traditional transportation problem. The basic idea of the algorithm is in adjusting the cost matrix of the Relaxed Transportation Problem (RTP), after each allocation (iteration) according to the changes in the supply and demand. The following steps illustrate the proposed algorithm:

Step1: Construct the Balinski RTP by relaxing the integer condition on y_{ij} ($y_{ij} = f_{ij}/m_{ij}$) and the unit transportation cost C_{ij} can be represented by (4).

$$C_{ij} = c_{ij} + f_{ij} / m_{ij} \qquad (4)$$

$$\text{Where} \quad m_{ij} = \begin{cases} \min(a_i, b_j) & if\ a_i, b_j > 0 \\ a_i & if\ b_j = 0 \\ b_j & if\ a_i = 0 \end{cases}$$

Step 2: For each row (column), determine a penalty by subtracting the smallest unit cost element in that row (column) from the next smallest unit cost element in the same row (column).

Step 3: Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the cell with the least cost in the identified row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out.

Step 4: If exactly one row (column) with positive supply (demand) remains uncrossed out, allocate this supply (demand) in the remaining uncrossed out cells with their unsatisfied demands (supply) of the uncrossed out column (row) and Stop. Otherwise go to step 5.

Step 5: Adjust the last cost matrix by recalculating the C_{ij} for the uncrossed out row (column) identified in step 3. Go to step 3.

Numerical Example

Consider a company with four factories in locations S_1, S_2, S_3 and S_4 which produce a specific type of product. There are six other locations D_1, D_2, D_3, D_4, D_5 and D_6 that receive this product as consumers. The supply S_i, the demand D_j, the cost f_{ij} for opening the route (i, j) and the unit cost c_{ij} for transporting one unit of the given product from the source i to the destination j are given in Table 1.

Table1: Cost matrix f_{ij}, c_{ij} [13]

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	11 , 0.69	16, 0.64	18, 0.71	17, 0.79	10, 1.7	20, 2.83	45
S_2	14, 1.01	17, 0.75	17, 0.88	13, 0.59	15, 1.5	13, 2.63	35
S_3	12, 1.05	13, 1.06	20, 1.08	17, 0.64	13, 1.22	15, 2.37	20
S_4	16, 1.94	19, 1.5	15, 1.56	11, 1.22	15, 1.98	12, 1.98	15
Demand	35	30	25	15	5	5	

Step 1: The Balinski RTP can be represented as in Table 2.

Table2: Balinski's RTP matrix with $C_{ij} = c_{ij} + f_{ij}/m_{ij}$

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	1.00	1.17	1.43	1.92	3.70	6.83	45
S_2	1.41	1.32	1.56	1.46	4.50	5.23	35
S_3	1.65	1.71	2.08	1.77	3.82	5.37	20
S_4	3.01	2.77	2.56	1.95	4.98	4.38	15
Demand	35	30	25	15	5	5	

Step 2: Calculation of the penalty of each row (P_i) and of each column (P_j) is shown in Table 3.

Table 3: (P_i) and (P_j)

	D_1	D_2	D_3	D_4	D_5	D_6	Supply	P_i
S_1	1.00	1.17	1.43	1.92	3.70	6.83	45	0.17
S_2	1.41	1.32	1.56	1.46	4.50	5.23	35	0.09
S_3	1.65	1.71	2.08	1.77	3.82	5.37	20	0.06
S_4	3.01	2.77	2.56	1.95	4.98	**4.38**	15	0.61
Demand	35	30	25	15	5	5		
P_j	0.41	0.14	0.13	0.32	0.12	**0.85**		

Step 3: From Table 3, the sixth column has the maximum penalty (0.85) and the minimum cost in this column is 4.38 in the cell (4, 6). So, the cell (4, 6) is allocated with the maximum possible value (5 units) which is the minimum of S_4 and D_6. Cross out the column D_6 corresponding to this *minimum*. The uncrossed out row is the fourth row, then change the supply S_4 to 10. The result of this step is allocating 5 units in the cell (4, 6). This means that x_{46} = 5.

Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.

Step 5: Since the uncrossed out row in step 3 is S_4, calculate the costs C_{4j} for row S_4 based on its new supply (10). The new cost matrix is presented in Table 4. Note that the costs in row S_4 only are changed due to changing S_4 from 15 to 10. Go to step 3.

Table 4: Cost Matrix from Step 4

	D_1	D_2	D_3	D_4	D_5	Supply	P_i
S_1	1.00	1.17	1.43	1.92	3.70	45	0.17
S_2	1.41	1.32	1.56	1.46	4.50	35	0.09
S_3	1.65	1.71	2.08	1.77	3.82	20	0.06
S_4	3.54	3.40	3.06	**2.32**	4.98	10	**0.74**
Demand	35	30	25	15	5		
P_j	0.41	0.14	0.13	0.32	0.12		

Step 3: From Table 4, the row S_4 has the maximum penalty (0.74) and the minimum cost in this row is 2.32 in the cell (4, 4), so allocate this cell with 10 units which is the minimum of D_4 and S_4 .Cross out the row S_4 corresponding to this *minimum*. The uncrossed out row is the fourth column. Change the demand D_4 to 5. The result of this step is allocating 10 units in the cell (4, 4). This means that x_{44} = 10.

Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.

Step 5: Since the uncrossed out column is D_4, calculate the costs C_{i4} for that column based on its new demand (5). The new cost matrix is presented in Table 5.

Table 5: Cost Matrix from Step 5

	D_1	D_2	D_3	D_4	D_5	Supply	P_i
S_1	1.00	1.17	1.43	4.19	3.70	45	0.17
S_2	1.41	1.32	1.56	3.19	4.50	35	0.09
S_3	1.65	1.71	2.08	4.04	3.82	20	0.06
Demand	35	30	25	5	5		
P_j	0.41	0.14	0.13	**0.85**	0.12		

Step 3: The result of this step is transporting 5 units through the route (2, 4). That means x_{24} = 5.

Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.

Step 5: Since the uncrossed out row is S_2, recalculate the cost C_{2j}. The result of this step is presented in Table 6.

Table 6: Recalculated costs of Step 5

	D_1	D_2	D_3	D_5	Supply	P_i
S_1	**1.00**	1.17	1.43	3.7	45	0.17
S_2	1.48	1.32	1.56	4.5	30	0.09
S_3	1.65	1.71	2.08	3.82	20	0.06
Demand	35	30	25	5		
P_j	**0.47**	0.14	0.13	0.12		

Step 3: The result of this step is transporting 35 units through the route (1, 1). That means $x_{11} = 35$, *the* S_1 would be equal to 10 and the crossed out column D_1.
Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.
Step 5: Since the uncrossed out row is S_1, recalculate the cost C_{1j} as presented in Table 7.

Table 7: Recalculated costs of Step 5

		D_2	D_3	D_5	Supply	P_i
S_1		2.24	2.51	3.7	10	0.27
S_2		1.32	1.56	4.5	30	0.24
S_3		1.71	2.08	3.82	20	0.37
Demand		30	25	5		
P_j		0.39	**0.52**	0.12		

Step 3: The result of this step is transporting 25 units through the route (2, 3). That means $x_{23}= 25$, *the* S_2 would be equal to 5.
Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.
Step 5: Since the uncrossed out row is S_2, recalculate the costs C_{2j} of row 2 as presented in Table 8.

Table 8: Recalculated costs of Step 5

		D_2		D_5	Supply	P_i
S_1		2.24		3.7	10	1.46
S_2		4.15		4.5	5	0.35
S_3		**1.71**		3.82	20	**2.11**
Demand		30		5		
P_j		0.53		0.12		

Step 3: The result of this step is transporting 20 units through the route (3, 2). That means $x_{32}= 20$, and D_2 will be adjusted to 10.
Step 4: Since there are more than one row (column) with positive supply (demand) uncrossed out, go to step 5.
Step 5: Since the uncrossed out column is D_2, recalculate the costs C_{i2} of column 2 as presented in Table 9.

Table 9: Recalculated costs of Step 5

		D_2		D_5	Supply	P_i
S_1		2.24		3.7	10	1.46
S_2		4.15		4.5	5	0.35
Demand		10		5		
P_j		**1.91**		0.80		

Step 3: The result of this step is transporting 10 units through the route (1, 2). Cross out D_2 and adjust S_1 to 0 as in Table 10.

Table 10: Recalculated costs of Step 5

		D_2		D_5	Supply	P_i
S_1				**3.7**	0	**1.46**
S_2				4.5	5	0.35
Demand				5		
P_j				0.80		

Step 4: Since there is only one column (D_5) with positive demand (5) uncrossed out, allocate 5 units to the cell (2, 5) and stop. The final allocation is shown in Table 11.

Table 11: Final allocation

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	35	10			0		45
S_2			25	5	5		35
S_3		20					20
S_4				10		5	15
Demand	35	30	25	15	5	5	

The total variable cost $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ of final allocation is 106.3, the total fixed cost $\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}y_{ij}$ is 108 and the total cost $\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + f_{ij}y_{ij})$ is 214.3. While the proposed algorithm finds an initial feasible solution with total cost equals 214.3, which is less than the initial feasible solution (the optimal RTP solution) given in (Balinski, 1961) gives total cost of 229.1.

Computational Results and Analysis

As many as 37 FCTP problems have been chosen, with different dimensions (ranging from 3x4 up to 17×17) from different references, including from OR library ("OR Library: Testcases for Transportation Problems, Fixed Charge Transportation Benchmark Problems,,") and (Adlakha et al., 2014). Table 12 shows a comparison between the objective functions of the approximate solutions using the proposed algorithm (OFPA) and the results of the approach (OFB) proposed in (Balinski, 1961), who adopted results from (Adlakha et al., 2010). The two approaches gave the same solutions for 7 problems viz., 8, 25, 26, 28, 29, 31 and 32. The approach gave better solutions for 4 problems viz., 9, 17, 35 and 36. In order to establish how close is the objective function of the approximate solution to the optimal solution of the FCTP (OFFCTP), using both the proposed algorithm and the approach in (Balinski, 1961), paired-sample t-test was carried out. The test was carried out on the results of 37 problems considered in Table 12. The results of the test are presented in Tables 13 and 14. It can be observed that the difference between the value of the objective function of the approximate solution by the proposed algorithm and the optimal solution of the FCTP is significantly lower than the difference between the proposed and value of the objective function of the approximate solution by the Balinski's approach (Balinski, 1961) and the optimal solution of the FCTP. Hence the proposed algorithm can be considered superior to that of Balinski's and provides a technique for finding the initial solution for the FCTP.

Table 12: Comparison between the objective functions of approximate solutions using the proposed algorithm and the Balinski's approach.

No.	Problem and Dimension	OFPA	OFB	OFFCTP	No.	Problem and Dimension	OFPA	OFB	OFFCTP
1	gr4×6	214.3	229.1	202.35	20	Ex(5)9x9 (5)	2666	2680	2042.50
2	ran10×10a	1808	1736	1499	21	EX(3)8x8(3)	2880	3001	2110.10
3	ran10×10c	14480	16490	13007	22	EX(2)8x8(2)	3741	4296	2854.90
4	ran10×12	3365	3260	2714	23	EX(5)8x8(5)	4139	4470	3297.20
5	ran12×12	2508	2683	2291	24	3×4	560	570	495
6	ran13×13	3390	3521	3252	25	4×5a	2590	2590	2305.40
7	ran16×16	4273	4333	3823	26	4×5b	3370	3570	2633.30
8	bk4×3	360	360	350	27	4×5c	680	780	653.30

9	ran17×17	1525	1464	1373	28	4×5d	9900	9900	8946.70
10	bal8×12	501	504.2	471.5	29	4×5e	1960	1960	1743.30
11	kow4×5	265	285	250	30	4×5f	315	320	305.80
12	Kawl4×5	335	345	335	31	4×5h	325	325	296.70
13	ran10×10bT	3491	3546	2672.80	32	4×5i	345	345	311.70
14	ran10×10cT	16591	16653	12544.7	33	3×4	30350	30455	29950
15	ran10×12T	2874	3289	2326.70	34	4×5	395	405	325.70
16	ran12×12T	2559	2691	1972.30	35	4×6	835	805	740
17	Ex(3)7x8	2545	2449	1970.60	36	4×5	3170	3150	2620.00
18	Ex(15)7x8(3)	2493	2510	1974.20	37	4×5	1350	1360	1350.00
19	Ex(6)7x9(1)	2248	2408	1894.40					

Table 13: Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair	Difference from OFFCTP to Proposed Algorithm	472.7338	37	700.42292	115.14882
	Difference from OFFCTP to Balinski Results	590.0851	37	867.61925	142.63573

Table 14: Paired Samples Test Results

			Difference from OFFCTP to Proposed and that of Balinski
Paired Differences	Mean		-117.35135
	Std. Deviation		346.78290
	Std. Error Mean		57.01076
	95% Confidence Interval of the Difference	Lower	-232.97453
		Upper	-1.72818
T			-2.058
Df			36
Sig. (2-tailed)			.047

Table 15: Comparison between proposed and Balinski's approaches.

Problem No.	Problem and Dimension	RTP of OFOB	RTP of OFOP	Optimal Solution of the FCTP	Reference
1	gr4×6	185	191.3	202.35	OR Library
2	ran10×10a	1252.4	1385.4	1499	OR Library
3	ran10×10b	2613.5	2715.1	3073	OR Library
4	ran10×10c	11203.1	12620.7	13007	OR Library
5	ran10×12	2426.2	2526.3	2714	OR Library
6	ran12×12	1826.5	1941.9	2291	OR Library
7	ran13×13	2691.4	2736.1	3252	OR Library
8	ran16×16	3116.4	3314.9	3823	OR Library
9	bk4×3	321.67	326.7	350	OR Library
10	ran17×17	1215.2	1374.5	1373	OR Library
11	bal8×12	451.2	473.6	471.5	OR Library
12	ran14×18	3016.9	3227.5	3712	OR Library
13	kow4×5	226	258.3	250	Adalkha
14	Kawl4×5	305	335.0	335	Adalkha

Table 15 shows that the values of the RTP obtained by the proposed approach (OFOP) lies between the values of the RTP given by Balinski's approach (OFOB)(Balinski, 1961) and the optimal solution of the RTP matrix, for some of the problems considered in OR library ("OR Library: Testcases for Transportation Problems, Fixed Charge Transportation Benchmark Problems,,") and (Adlakha et al., 2014). Also, all the values do not penetrate the optimal values of such problems. Hence, the RTP values given by the proposed algorithm can be considered as a lower bound – as a reference - for the optimal solution of FCTP instead of using the RTP value given by the optimal solution of RTP matrix as mentioned in (Adlakha et al., 2010).

Conclusion

This paper presented a heuristic approach for finding an approximate solution used as a lower bound for the optimal solution of FCTP. This heuristic approach has been applied to a set of problems and the results indicate that it is significantly better. The RTP value using this algorithm can be considered as a better lower bound to the optimal solution of FCTP compared to the RTP value obtained by Baliniski's approach (Balinski, 1961). In addition, the proposed algorithm is simple and computationally feasible as compared to the algorithm presented in (Adlakha et al., 2014) which is dealing with a non-linear formulation of the problem.

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