Planning of the Hierarchical Production in the Forestry Industrial Sector

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Abstract

This chapter proposes a new and efficient heuristic algorithm to solve the problem of hierarchical production of limber in sawmills. The proposed solution is based on Mixed-Integer Linear Programming (MILP), using Benders' decomposition and Lagrangean relaxation techniques The proposed methodology achieves a higher computational efficiency than the state of the art for the solution of this kind of problems.

Key Words: *Planning, Hierarchical, Production, Forestry, Industrial Sector.*

Introduction

Mexican companies are immerse in a world-wide competition González S. F. (1995, 2004, 2005, 2008), states that in a globalized market, Mexican companies will find difficult to remain in the market to distribute its products, unless they modernize to a competent infrastructure. This modernization includes material and human resources, and a control system that allow them to operate with a high degree of efficacy and efficiency: the requirements in our current reality.

Under those circumstances, the company requires an optimal production planning and scheduling to gain competitive advantage. This can be done assuming that goods and/or services provided by the company are certified and competitive in price and quality.

The production-planning problem Aardal K. et al (1990) addressed in this contribution is classified as NP-Complete. The goal of this problem is to plan and schedule production over time Bitran G.R., Hax A.C. (1977), González S.F. et.al (1996, 2011). Production planning has fundamental objectives: the planning function determines the requirements sources, the current point in time determines the planning horizon and the order of demand satisfaction. The scheduling function determines how the available production sources act, locating the individual products provided to consumers at a minimum production cost. The problem is to achieve a production volume for the analysis period at high efficiency levels. Achieving this goal guarantees the company's permanence in the global markets. This contribution addresses this problem in the framework of a limber production sawmill; we present its mathematical model, a solution algorithm and the results of the production plan.

Problem Statement

On of the most important economic activities of the state of Michoacan, Mexico is the industrialization of the forestry resources. The first industrialization stage from the mechanical point of view of this resource is sawmills. The stages that form this process González Santoyo F. (1987) are: Reception and classification of raw material, edging, multiple head remover, classification table and storage. The problem´s representative variables are: raw material, machine time, processing time availability, supplies, labor, electricity, maintenance and sawn products market.

One problem found in the process is that logs present different diameters, lengths and uniformity ranges, so they need to be classified according to those variables. This enables a more efficient operation and the choice of an optimal cut schedule for the main saw and other stages in the process.

In the state of Michoacan, Mexico, the main problem is the log length. Logs are supplied in lengths of 8, 10, 12, 14, 16, 18, 20, and 22 feet, with diameters from 12 to 30 inches. Commercial measures are ½, ¾, 1½, 3, 3½ inches thick, 8, 10, and 12 inches wide. These conditions allow us to deal with this problem using Mixed-Integer Linear Programming (MILP). We have 8 kinds with 1, 2, and 4 families, which means to have 160 elements of sawn lumber in the company.

Using MILP, production is aggregated in families and families in types of products Bitran G.R., Haas E.A., Hax A.C. (1981) This aggregation structure [González S.F. (1995)], lies in the research line known as hierarchical production planning. Planning of the original production is divided in a hierarchy of sub problems, where a structuring production plan considers that individual parts and final products are aggregated.

The grouping criterion follows Bitran (1981), using the description provided by [González S.F. (1995)]: Items are final products, required by the market in a time unit, types of products are groups of items that have similar production costs, present the same demand model and the same production range. Types are characterized according to log length. Families of products are represented by a set of items that share a common characteristic. In this case this characteristic is mean width.

The general MILP problem is characterized as a large-scale problem. The hierarchical planning problem will be solved using MILP and a heuristic algorithm, based on Bender´s decomposition theory Aardal K., Larsson T. (1990), González S.F. et.al (1996).

Mathematical Model

The hierarchical planning problem (PS) can be formulated as follows:

$$
\begin{aligned}
& Z_{ps} Min. \sum_{t} (C_t O_t + \sum_{i} h_{it} I_{it}) + \sum_{j} \sum_{t} S_{jt} X_{jt} \\
& \sum_{j \in T(i)} FP - P = 0, \forall i,t \\
& \sum_{l} K_i P_{it} - O_t \leq r_t, \forall t, \\
& \sum_{j \in T(i)} FI_{jt} - I_{it} = 0, \forall i,t \\
& FP_{jt} + FI_{j,t-1} - FI_{jt} = d_{jt}, \forall j,t \\
& FP_{jt} - m_{jt} X_{jt} \leq 0, \forall j,t \\
& O_t, P_{lt}, I_{it}, FP_{jt}, FI_{jt} \geq 0, \forall i,j,t \\
& X_{jt} \in \{0,1\}, \forall j,t
\end{aligned}
\qquad (1)
$$

where t is current the time period, C_t the cost of one hour of extra time, h_{it} is the inventory cost for items of type i, S_{jt} is the preparation cost per family j, d_{it} (d_{jt}) is the demand of item i (family j), K_i is the required production time for i, T(i) is the set of families that belong to type i, m_{jt} is the production amount for family j, and r_t is the available production time. The model´s decision variables re: C is the number of extra time hours for production at time t, I_{it} (FI_{jt}) is the inventory of type i (family j), P_{it} (FP_{jt}) is the production amount of type I (family j), X_{jt} is a 0-1 variable that indicates the update of family j.

The solution process is based on Bender´s decomposition techniques. Complex variables in PS are X_{jt}, FI_{jt}, and FP_{jt}; these variables allow us to structure the problem on a type and family levels. For this variable partition, the Bender´s sub problem $PSUB_T$ can be stated as:

$$\begin{aligned} & Z_{PSUB}(t) = Min.C_t O_t + \sum_i h_{it} I_{it} \\ & P = \sum_{j \in T(i)} FPjt \ , \forall i \\ & \sum_i K_i P_{it} - O_t \le r_t \\ & I_{it} = \sum_{j \in T(i)} FI_{it}, \forall i \\ & O_t, I_{it}, P_{it} \ge 0, \forall i \end{aligned} \tag{2}$$

An upper bound on the optimal value is provided by the sub problem, when the constant $\sum_j \sum_t S_{jt} X_{jt}$, is added to $\sum_t Z_{PSUB}(t)$, becomes a PS constraint. The sub problem has unique feasible primal solution, which is found by inspection as follows:

$$\begin{aligned} & P_{it} = \sum_{j \in T(i)} FP_{jt}, \forall i, t \\ & I = \sum_{j \in T(i)} FI_{jt}, \forall i, t \\ & O_t = Max.\left\{0, \sum_i K_i (\sum_{j \in T(i)} FP_{jt}) - r_t \right\}, \forall t \end{aligned} \tag{3}$$

Let U_{it}, V_t, W_{it}, be the dual variables associated to the primal constraints, given in the formulation $PSUB_t$. The $PSUB_t$ dual problem $DPSUB_t$ can be stated as:

$$\begin{aligned} & Z_{DPSUB}(t) = Max.\sum_i U_{it} (\sum_{j \in T(i)} FP_{jt}) + V_t r_t + \sum_i W_{it} (\sum_{j \in T(i)} FI_{it}) \\ & U_{it} + K_i V_t \le 0, \forall i \\ & -V_t \le C_t \\ & W_{it} \le h_{it}, \forall i \\ & V_t \le 0 \\ & U_{it}, W_{it}, \forall i \end{aligned} \tag{4}$$

The solution of the dual problem, obtained by inspection, is:

$$
\begin{aligned}
W_{it} &= h_{it,\forall i,t} \\
V_t &= \begin{Bmatrix} -C_t & \text{si} & O_t > 0, \forall t \\ 0 & \text{si} & O_t = 0, \forall t \end{Bmatrix} \\
U_{it} &= -K_i V_t, \forall i,t
\end{aligned} \tag{5}
$$

DPSUB has multiple solutions, since PSUB is a degenerate solution. An alternative is to make U_{it} y W_{it} zero, if $\sum_{j \in T(i)} FP_{jt}$ y $\sum_{j \in T(i)} FI_{jt}$ are zero, respectively.

In the case where $O_t = 0$, we can obtain the Bender´s cut $Z_t \geq \sum_i h_{it} (\sum_{j \in T(i)} FI_{jt})$, where Z_t, is an auxiliary variable from Bender´s master problem.

These kinds of cuts are favorable and are included in the master problem. Let U^*_{it} y V^*_t be the dual solutions corresponding to the case $O_t > 0$. Let us assume PSUB has been solved for a sequence of given variables and that the $O_t > 0$ occurs at least once for the periods $T^* \subseteq \{1,\ldots,T\}$. The master problem $PM1$ can be stated as:

$$
\begin{aligned}
&Z_{PM} = Min. \sum_j \sum_t S_{jt} X_{jt} + \sum_t z_t \\
&z_t \geq \sum_i U^*_{it} (\sum_{j \in T(i)} FP_{jt}) + V^*_t r_t \\
&+ \sum_i h_{it} (\sum_{j \in T(i)} FI_{jt}), t \in T^* \\
&z \geq \sum_i h_{it} (\sum_{j \in T(i)} FI_{jt}), \forall t \\
&FP_{jt} + FI_{j,t-1} - FI_{jt} = d_{jt}, \forall j,t \\
&FP_{jt} - m_{jt} X_{jt} \leq 0, \forall j,t \\
&O_t, P_{it}, FP_{jt}, FI_{jt}, I_{it} \geq 0, \forall i, j, t \\
&X_{jt} \in \{0,1\}
\end{aligned} \tag{5}
$$

A variable substitution allows us to reformulate the master problem $PM2$ as follows:

$$
\begin{aligned}
&Z_{PM} = Min. \sum_j \sum_t S_{jt} X_{jt} + \sum_i \sum_t h_{it} (\sum_{j \in T(i)} FI_{jt}) + q_t \\
&q_t \geq \sum_i U^*_{it} (\sum_{j \in T(i)} FP_{jt}) + V^*_t r_t, t \in T^* \\
&q_t \geq 0, \forall t \\
&FP_{jt} + FI_{j,t-1} - FI_{jt} = d_{jt}, \forall j,t \\
&FP_{jt} - m_{jt} X_{jt} \leq 0, \forall j,t \\
&O_t, P_{it}, FP_{jt}, FI_{jt}, I_{it} \geq 0, \forall i, j, t \\
&X_{jt} \in \{0,1\}, \forall j,t
\end{aligned} \tag{6}
$$

The master problem is a relaxation of PS, where Z_{PM} provides a lower bound to Z_{PS}. [González S.F. (1995)], suggests the use of a Lagrangean relaxation as a strategy to solve problems of PM2. By relaxing Bender´s cuts with Lagrangean multipliers λ_t, the objective function gets the form:

$$Min.$$
$$\sum_j \sum_t S_{jt} X_{jt} + \sum_i \sum_t h_{it}\Big(\sum_{j\in T(i)} FI_{jt}\Big) + \sum_t q_t + \sum_{t\in T^*} I_t(g_t(FP,FI) - q_t \tag{7}$$
$$where$$
$$g(FP,FI) = \sum_i U_{it}^*\Big(\sum_{j\in T(i)} FP_{jt}\Big) + V_t^* r_t$$

Each q_t is a non-negative continuous variable that does not appear in the rest of the constraints. Assuming $\lambda_t \leq 1$, $t\in T^*$, q_t´s coefficient becomes non-negative, and q_t´s optimal value approaches zero as the objective function approaches its minimum.

PM2´s Lagrangean relaxation $LRPM$ can be stated as:

$$\begin{aligned} Z_{LRPM}(\lambda) &= Min.\sum_j \sum_t S_{jt} X_{jt} + \sum_i \sum_t h_{it}\Big(\sum_{j\in T(i)} FI_{jt}\Big) \\ &+ \sum_{j\in T(i)} \lambda_t g_t(FP,FI) \\ & FP_{jt} + FI_{j,t-1} - FI_{jt} = d_{jt}, \forall j,t \\ & FP_{jt} - m_{jt} X_{jt} \leq 0, \forall j,t \\ & O_t, P_{it}, FP_{jt}, FI_{jt}, I_{it} \geq 0, \forall i,j,t \\ & X_{jt} \in \{0,1\}, \forall j,t \end{aligned} \tag{8}$$

By relaxing PM, the resulting problem LRPM, is separated by families in a set of problems of Economic Lot Size without capacity constraint. This kind of problems can be solved efficiently by dynamic programming [Wagner, Whitin (1958)]. From the Lagrangean Duality theory for Integer Programming, Z_{LRMP} is a lower bound to Z_{PM}. The largest available lower bound, D, is obtained by the solution to:

$$\begin{aligned} Z_D &= Max. Z_{LRPM}(\lambda) \\ 0 &\leq \lambda \leq 1, t \in T^* \end{aligned} \tag{9}$$

Which is the Lagrangean Dual with respect to the relaxed Bender´s cuts. This terminology is used since since $\lambda=\{\lambda_{tt}\}$ plays a similar roll LRPM with Lagrangean Multipliers normally used in the continuous problem. The dual objective function $Z_{LRPM}(\lambda)$ is continuous, concave, piecewise linear, and sub-differentiable.

A standard procedure to solve the dual problem is the sub-gradient optimization algorithm, which generates dual solutions according to the following rule.

$$\lambda_t^{l+1} = \lambda_t^l + \theta\gamma_t^l \qquad ,t\in T^*, \quad l = 0,1,\ldots \tag{10}$$

where $\gamma=\{\gamma_t\}$ is a sub-gradient of $Z_{LRPM}(\lambda)$ for a particular value of λ and Θ_l is the step size.

Solution Algorithm

1. Assume I T^*I Bender's cuts have been generated and that PM2 is solved using Lagrangean Relaxation and Subgradient Optimization.
2. Vector λ_t is updated a number of times determined by the equation.

$$\lambda^{l+1} = \lambda_t^l + \theta_1 \gamma_t^l, \quad t \in T^*, l = 0,1,...$$

For a particular value of λ, $\gamma = \{\gamma_t\}$ is a subgradient of $Z_{lrpm}(\lambda)$ and Θ_1 is the step size. At every point in time LRPM is solved and lower bound to Z_{ps} is determined.

3. The values of variables Xjt, FIjt, FPjt, obtained from LRPM solution, are used to execute PSUB, while the set of updated variables is improved for the heuristic exchange.
4. The optimal dual variables sent to PSUB are based on new Bender's cuts. Nonetheless, a cut may already be included in PM2. A previously generated cut cannot be included again in PM2.
5. An upper bound for Z_{ps} is obtained by a heuristic exchange for PSUB's solution; this value depends on the lower bound.

In the next iteration, the subgradient procedure for PM2 continues from the last solution to LRPM. The procedure is initialized by initial values $\{X_{jt},\}$, which indicate the amount of product for family j at time t. The algorithm continues until a given number of Bender's subproblems have been solved or the difference between the upper and lower bounds is small enough, according to a previously established parameter. Dual variables U_{it}, W_{it} are part of the input to LRPM's objective function. These variables can be interpreted as production and storage costs for families that belong to a certain type of product.

Study Case

The proposed algorithm was used to solve problems that included 1, 2, and 4 families from the 8 different types in the problem. 160 different products were included (commercial sawn wood products), which are being produced in the sawmill under the current supply conditions. A family's model includes 5 different commercial sizes of sawn products; 4 families include 20 different sizes for each one of the types they produce.

Each production plan specifies the commercial size, its location within the family, and its type. The size contributes to the objective function to minimize cost; to satisfy demand, the production amount is specified and the inventory level at the planning horizon (assumed to be 1 year). Since this is a dynamic analysis process, it allows the manager to define the plant's operation form. This section presents an example with the results of the production plan for type 1, characterized by 8´ length, 4 families, and different commercial diameters (indicated in the following tables). The plan information is expressed in P.U. (per unit).

Table 1. Commercial sawn wood sizes

Size	Size	Size	Size
½´´x6´´x8	½´´x8´´x8	½´´x10´´x8	½´´x12´´x8
¾´´x6´´x8	¾´´x8´´x8	¾´´x10´´x8	¾´´x12´´x8
11/2´´x6´´x8	11/2´´x8´´x8	11/2´´x10´´x8	11/2´´x12´´x8
2´´x6´´x8	2´´x8´´x8	2´´x10´´x8	2´´x12´´x8
31/2´´x6´´x8	31/2´´x8´´x8	31/2´´x10´´x8	31/2´´x12´´x8

Results

Table 2. Production Plan Type I

	Family 1			Family 2			Family 3			Family 4		
T	D	P	I	D	P	I	D	P	I	D	P	I
1	40	125	0	40	90	0	65	65	0	70	120	0
2	60	0	85	50	0	50	120	120	0	50	0	50
3	25	0	25	50	90	0	120	145	0	100	190	0
4	65	65	0	40	0	40	25	0	25	90	0	90
5	120	120	0	70	120	0	60	120	0	80	120	0
6	120	145	0	50	0	50	60	0	60	40	0	40
7	25	0	25	100	190	0	85	125	0	60	85	0
8	60	120	0	90	0	90	40	0	40	25	0	25
9	60	0	60	80	120	0	40	90	0	65	65	0
10	85	165	0	40	0	40	50	0	50	120	120	0
11	40	0	80	60	85	0	50	90	0	120	145	0
12	40	0	40	25	0	25	40	0	40	25	0	25

T= time (months), D = Demand, P = Production Volume (units), I = Inventory Level.

The product type and family for Plan I, characterized by 8´ (feet) long in different width and thick sizes are shown in the following table.

Table 3. Product Type and Family for Plan I

FAMILIES	PRODUCT	T (min)	COST ($)
1	(3/4” x 6” x 8”)		
2	(1/2” x 8” x 8”)	.826	3,801.00
3	(2” x 10” x 8”)		
4	(3/4” x 12” x 8”)		

These results allow the manager to make efficient and effective decisions, associated to the plan’s operation cost, the product volume and type of commercial sizes the market demands. It is also important an appropriate management of an minimal inventory system capable of satisfying the market’s demand in contingency conditions with the least financial resources invested in it.

Conclusions

From the resulting production plan, we conclude that for large-scale problems, the proposed algorithm is computationally efficient. The solution is obtained in 4 iterations. The proposed methodology presents a practical flexibility for efficient decision making in the company. Thus, the algorithm can be easily deployed on a personal computer, presenting a greater flexibility than the currently available commercial software to solve this kind of problem.

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